8.1 Complex Numbers

Question Paper

Course	CIE A Level Maths
Section	8. Complex Numbers
Topic	8.1 Complex Numbers
Difficulty	Hard

Time allowed: 60

Score: /44

Percentage: /100

Question 1

Given that $z_1 = 1 - 2\text{i}$ and $z_2 = -3 + 5\text{i}$, work out the following:

(i) $\text{Re}(z_2 - z_1)$

(ii) $\text{Im}(z_1 z_2)$

(iii) $\left(\frac{z_1}{z_2}\right)^*$

For part (iii) give your answer in the form $a + b\text{i}$, where a and b are real numbers.

[6 marks]

Question 2

The complex number z satisfies the equation $(2 + 5\text{i})(z + 2\text{i}) = -7 - 32\text{i}$.
Find z, giving your answer in the form $a + b\text{i}$, where a and b are real numbers.

[4 marks]

Question 3a

(a) Given that $z_1 = a - 6\mathrm{i}$, $z_2 = 1 + b\mathrm{i}$, and $z_1 z_2 = -17 - 9i$, where a and b are real numbers, find the possible values of a and b.

[4 marks]

Question 3b

(b) Using your answers to part (a), write down values for c and d that will satisfy the equation

$$-(3 + \mathrm{i})(c + d\mathrm{i}) = -17 - 9\mathrm{i}$$

[2 marks]

Question 4

The equation $z^2 + bz + 18 = 0$, where $b \in \mathbb{R}$, has distinct non-real complex roots.
Find the range of possible values of b.

[3 marks]

Question 5

Given that $-3 + 2\text{i}$ is one of the roots of the quadratic equation $z^2 + bz + c = 0$, where b and c are real constants, find the values of b and c.

[4 marks]

Question 6

$f(z) = z^2 + 2iz - 10$

(i) Show that $f(z)$ can be rewritten in the form $(z + ai)^2 + b$, where a and b are real numbers to be found.

(ii) Hence find the solutions to the equation $f(z) = 0$.

[4 marks]

Question 7a

(a) Show that $\alpha = -1 + 4i$ is a root of the cubic equation

$$z^3 + 5z^2 + 23z + 51 = 0$$

[3 marks]

Question 7b

(b) Find the other two roots of the cubic equation in part (a), being sure to show clear algebraic working.

[3 marks]

Question 8

$f(z) = z^3 + z^2 + cz + d$, where c and d are real numbers.

Given that 3 and $-2 - 3i$ are roots of the equation $f(z) = 0$, find the value of c and the value of d.

[5 marks]

Question 9

For a complex number z, a square root of z is a complex number $x + \mathrm{i}y$ where x and y are real numbers and where

$$(x + \mathrm{i}y)^2 = z$$

By expanding $(x + \mathrm{i}y)^2$ and solving the resultant equation for x and y, determine the two square roots of the complex number $z = -21 - 20\mathrm{i}$.

[6 marks]